Name: Feedback

SA367 · Mathematical Models for Decision Making　　　　Spring 2022 · Uhan

Quiz 4 – 2/22/2022

Instructions. You have 15 minutes to complete this quiz. You may not use any other materials (e.g., notes, homework, website).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1a	0.5	
1b	0.5	
2	3	
Total		/ 40

Start with Problem 2 on the reverse side.

Problem 1. Suppose you solved the dynamic program you gave in Problem 2 by solving the corresponding shortest/longest path problem. The algorithm you used outputs (i) the length of a shortest/longest path and (ii) the nodes in a shortest/longest path.

a. How would you use this output to determine the maximum probability that the overall GPS system will work properly? Give a hypothetical example if it helps.

Make sure to specify how you need to transform the length of a shortest/longest path in order to determine the maximum probability that the overall GPS system will work properly.

b. How would you use this output to determine how many spare units should be added to each subsystem? Give a hypothetical example if it helps.

Make sure to specify how you need to transform the change in state from one stage to the next in order to determine how many spare units should be added to each subsystem.

Problem 2. The next generation GPS system developed by SPAWAR requires three subsystems to all function properly. To increase the reliability of the system, spare units may be added to each subsystem. It costs \$10,000 to add a spare unit to subsystem 1, \$30,000 to subsystem 2, and \$20,000 to subsystem 3. The probability that each system will work, depending on the number of added spares, is given below:

	Probability that a subsystem works		
Number of added spares	Subsystem 1	Subsystem 2	Subsystem 3
0	0.85	0.60	0.70
1	0.90	0.85	0.90
2	0.95	0.95	0.95

Note that a maximum of two spares may be added to each subsystem. The problem is to maximize the probability that the overall GPS system will work properly, given that \$60,000 is available for spare units.

Formulate this problem as a dynamic program by giving its shortest/longest path representation. In particular:

- define the meaning of each stage,
- define the meaning of the nodes/states in each stage,
- draw the directed graph (nodes and edges),
- specify the edge lengths,
- specify the source and target nodes, and
- specify whether the goal is to find a shortest or longest path.

Hint. Note that the cost of a spare unit is different for each subsystem. Also, note that the overall system works if all 3 subsystems work, so

$$\Pr\{\text{overall system works}\} = \Pr\{\text{subsystem 1 works}\} \cdot \Pr\{\text{subsystem 2 works}\} \cdot \Pr\{\text{subsystem 3 works}\}$$

Many of you struggled with this problem. Here are some notes and hints that will hopefully help you approach the problem successfully:

- Note that you want to maximize $\Pr\{\text{overall system works}\}$, which can be computed as stated in the hint above. For example, if 1 spare is added to each subsystem, the probability that the overall system works is $(0.90)(0.85)(0.90) = 0.6885$.

  Many of you used 1 minus the probabilities in the table above. Note that this is not necessary, and incorrect.

- Note that a maximum of 2 spares may be added to each subsystem. More than 2 spares total may be added to the overall system.

- Most of you formulated a DP in which each stage corresponds to adding spares to a subsystem. This is correct. For the states, consider using the budget remaining for spare units (i.e. 0, 1, 2, 3, 4, 5, 6). You can limit each subsystem to 2 spares by including the correct number of edges from each node.